For different laminar sublayer forms of Eq. (4), the range of constants γ_v that lead to *nonsingular* behavior of the model are

$$a = 0, \quad \epsilon'' = C_\mu^{1/2} C_{\epsilon 2} \epsilon^{3/2}, \quad \gamma_v < 46.87$$

$$a = 1, \quad \epsilon'' = C_\mu C_{\epsilon 2} K \epsilon, \quad \gamma_v < 23.62$$

$$a = 2, \quad \epsilon'' = C_\mu^{3/2} C_{\epsilon 2} K^2 \epsilon^{1/2}, \quad \gamma_v < 18.80$$

$$a = 3, \quad \epsilon'' = C_\mu^2 C_{\epsilon 2} K^3, \quad \gamma_v < 16.77$$

Conclusion

For large values of γ_v the model may exhibit singular behavior. In the form of the RNG K-ϵ model that avoids the use of explicit wall functions, $a = 1$, so γ_v must be smaller than 23.62 to avoid singularities.

Acknowledgments

This work was supported by NASA Lewis Research Center under Contract NAS3-26702 and the Office of Naval Research under Contract N00014-92-C-0089.

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Precision Requirement for Potential-Based Panel Methods

James K. Nathman*
Analytical Methods Inc., Redmond, Washington 98052

Introduction

MORINO and Kuo[1] obtained good numerical results from a potential-based panel method for very thin wings (t/c = 0.1%) but small panel number ($N < 14$) while recognizing their matrix equation was nearly singular. Katz and Plotkin[2] and Yon et al.[3] experienced numerical difficulties from a similar approach when they analyzed airfoils with large panel number ($N > 40$) and trailing-edge angle less than 0.25 rad. The diverse behavior described in Refs. 1 and 3 can be explained by the precision of the method. Specifically, the errors in lift and trailing-edge loading observed in Ref. 3 can be reduced by a factor of 100 by replacing the constant 0.15916 with a more exact representation of $1/2\pi$.

Discussion

Program number 8 of Appendix D of Ref. 2 solves for the perturbation potential ϕ through the boundary integral equation,

$$(1/2)\,\phi(p) + \frac{1}{2\pi}\int_{S-p} \phi n \cdot \nabla(\ell n\ r)\ \mathrm{d}S = \frac{1}{2\pi}\int_S \frac{\partial\phi}{\partial n}\ell n\ r\ \mathrm{d}S \qquad (1)$$

where p is a point on the boundary contour S, and r is the distance from p to the arc dS where the normal n points into the flow. The normal derivative, $\partial\phi/\partial n$, is known on the airfoil surface. That is, the code solves an external Neumann problem using a potential-based integral equation. This theory description is similar to that of Ref. 1 but not to those of Refs. 2 and 3 because program number 8 does not solve a Dirichlet problem inside the airfoil. Dirichlet boundary conditions applied to a fictitious flow inside the airfoil are relevant to neither the derivation nor the application of Eq. (1).

The integral equation becomes a matrix equation when the airfoil and wake are discretized by a set of panels and the potential is described by a finite set of values. That is,

$$[A]\ \{\phi\} = [B]\left\{\frac{\partial\phi}{\partial n}\right\} \qquad (2)$$

The $1/2\pi$ factor on the left side of Eq. (1) is approximated by 0.15916 ($\pi = 3.1415$) in Ref. 2. Replacing π by $\pi + \varepsilon$ in Eq. (1), where ε is the error due to roundoff or truncation, and multiplying by $1 + \varepsilon/\pi$ result in

$$1/2\left(1+\frac{\varepsilon}{\pi}\right)\phi(p) + \frac{1}{2\pi}\int_{S-p}\phi n \cdot \nabla(\ell n\ r)\ \mathrm{d}S = \frac{1}{2\pi}\int_S \frac{\partial\phi}{\partial n}\ell n\ r\ \mathrm{d}S \qquad (3)$$

The corresponding matrix equation is

$$\left[A + \frac{\varepsilon}{2\pi}I\right]\{\phi\} = [B]\left\{\frac{\partial\phi}{\partial n}\right\} \qquad (4)$$

where I is the identity matrix. The solution of Eq. (4) is the sum of the potential for zero truncation and an error term found by

$$[A]\ \{\phi\}_{\text{error}} = -\frac{\varepsilon}{2\pi}\{\phi\}_{\varepsilon=0} \qquad (5)$$

The determinant of A approaches zero as the airfoil thickness goes to zero. This will amplify seemingly insignificant truncation into significant error. Roundoff error will always be present, but the use of a potential solution with small error in Eq. (5) will establish when the error is significant, e.g., when the lift is in error by 1%.

Application of Eq. (5) to several airfoils with small trailing-edge angle ($h/a < 0.25$) and many panels ($N > 40$) led to the observation that the circulation is approximately related to these two parameters by

$$\left(C\frac{h/a}{N} + \frac{\varepsilon}{2\pi}\right)(\phi_u - \phi_l) = O(h/c) \qquad (6)$$

where C is a constant determined numerically from the ratio of the exact circulation to the error in circulation found from Eq. (5). When the kinematic equations for the control points are augmented by the Kutta condition to form the matrix A, the matrix I is modified accordingly. Alternate forms for the Kutta condition can increase the value of C, leading to less sensitivity to truncation error.

The analysis was verified by application of the code of Ref. 2 to the airfoils presented in Ref. 3. The code was modified to specify π consistently as 3.1415 or 3.141593, whereas the published code used different values for separate expressions. Calculations were made in 32-bit precision on a Silicon Graphics Indigo.

Figure 1 compares the pressure distribution from the two values of π for an airfoil with a cusped trailing edge. The trailing-edge angle, 0.018, is nonzero because of the finite trailing-edge panels. With π consistently defined as 3.1415, the pressure from the modified code is indistinguishable from that of Ref. 3. The more exact code agrees with the theoretical solution drawn on the figure, including a well-behaved pressure distribution at the trailing edge. One measure of the accuracy of the solution is the pressure difference at the trailing edge, which should approach zero as panel size decreases. The difference in pressure of the upper and lower trailing-edge panels is 0.01 for $\pi = 3.141593$ compared with 0.8 for $\pi = 3.1415$. The original code has 16% excess circulation. The error

Received June 2, 1993; revision received Oct. 5, 1993; accepted for publication Oct. 13, 1993.

*Director, VSAERO Development. Member AIAA.

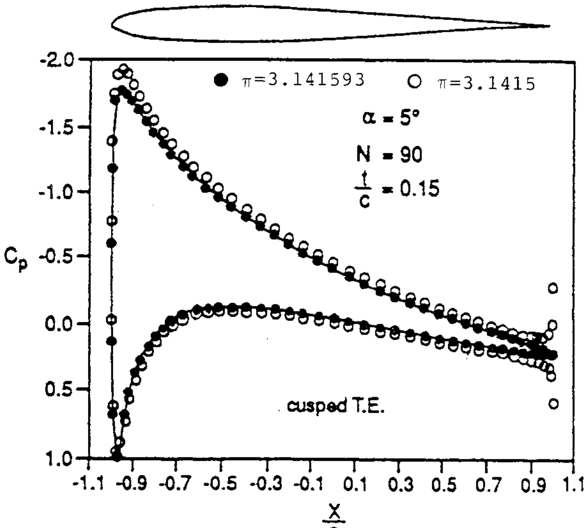

Fig. 1 Pressure distribution on an airfoil with a cusped trailing edge.

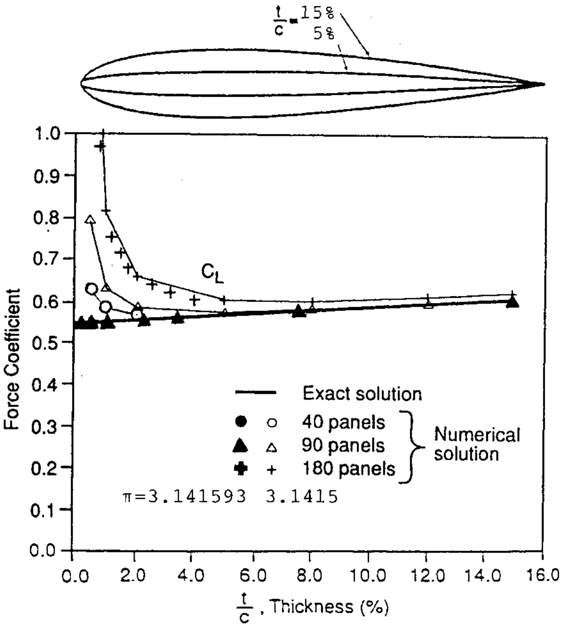

Fig. 2 Lift force vs airfoil thickness ratio.

predicted from linear analysis, Eq. (5), is 14% whereas Eq. (6) predicts an error of 16% based on a value of C of 0.55.

The lift coefficient is plotted vs airfoil thickness in Fig. 2 for airfoils with wedge trailing edges. The results for $\pi = 3.1415$ with 90 and 180 panels are indistinguishable from those of Ref. 3 and deviate markedly from theory for less than 5% thickness. The lift for 40 panels differs slightly from Ref. 3 because of error in integrating the pressure distribution. The more exact code is within 1% of the theoretical lift for the 40, 90, and 180 panel cases, including the thinnest airfoil indicated ($t/c = 0.25\%$, $h/a = 0.005$). The trailing-edge loading vs trailing-edge angle, Fig. 3, shows a consistent tendency for the original code to have positive loading from excessive circulation. The more exact code yields trailing-edge pressure differences near zero for the 40, 60, and 90 panel cases. Only the last is shown for clarity.

As seen in Fig. 2, for a thickness of 0.5% ($h/a = 0.01$) and 90 panels the error in π (–0.000093) in the original code produces an error in lift of 50%. This agrees with Eq. (6) given a value of C of 0.4 whereas Eq. (5) predicts 33%. For the corrected code ($\varepsilon/\pi = 10^{-7}$), the ratio $N/(h/a)$ must be less than approximately 80000 if the lift is to be accurate to 1%. For example, if 90 panels were used, the trailing-edge angle could be as small as 0.002 if all other calculations were exact.

VSAERO[4] (version E.4), a three-dimensional panel method used by the industry, was applied without special trailing-edge

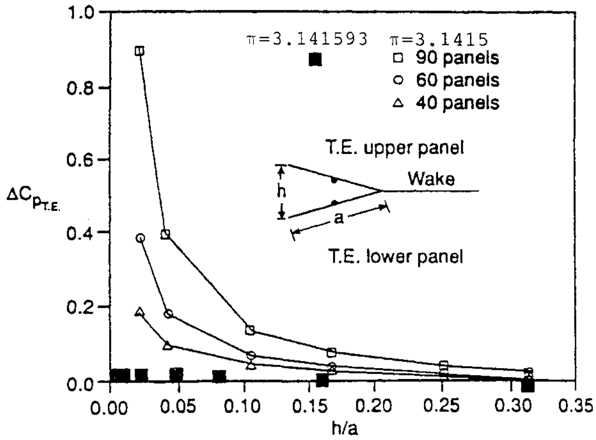

Fig. 3 Pressure difference between the trailing-edge upper and lower panels vs trailing-edge angle.

treatment to aspect ratio 20 wings, each with a cusped or wedge airfoil. Ten spanwise divisions were used with 90 chordwise panels. Lift and trailing-edge pressure difference at the wing centerline behaved the same as the more exact two-dimensional code for trailing-edge angles as low as 0.005. Sixteen spanwise divisions were needed for the 0.25% thick wedge airfoil to maintain the desired accuracy. Truncating π to 3.1415 increased the lift of the 1% thick wedge airfoil from 0.52 to 0.61 and trailing-edge loading from 0.01 to 0.96, close to the two-dimensional effect.

Potential-based panel methods require numerical precision in proportion to the ratio of the number of panels to the trailing-edge angle for accurate lift calculation. Based on results in two and three dimensions for airfoils with cusped and wedge trailing edges, it is concluded that single precision is sufficient for trailing-edge angles as low as 0.005. The problem studied in Ref. 3 is the result of imprecise specification of the value of π and is seldom encountered in computer codes used by the industry.

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Comparison of Finite Element and Finite Volume Methods for Incompressible Viscous Flows

K. Nakajima* and Y. Kallinderis†

University of Texas at Austin, Austin, Texas 78712

I. Introduction

FINITE element and finite volume methods are widely used as general appoximation methods in the area of computational fluid dynamics. A direct comparison between the two approaches

Received June 17, 1993; revision received Oct. 12, 1993; accepted for publication Oct. 12, 1993.

*Graduate Research Assistant; currently at Department of Aerospace Engineering and Engineering Mechanics, Mitsubishi Research Institute, Inc., Japan.

†Associate Professor. Member AIAA.